

# **An Analysis of the Determinants of the iTraxx CDS Spreads using the Skewed Student's t AR-GARCH Model**

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## **Abstract**

This paper examines the volatility clustering behaviour between the changes in the iTraxx 5-year Europe Credit Default Swap (CDS) index and its theoretical determinants using the AR-GARCH model with the skewed student's t distribution. Financial time series are known to be conditionally heteroskedastic, fat-tailed, and often skewed. It is demonstrated that the AR-GARCH model with the skewed student's t marginal density function provides both better in-sample fit and more accurate out-of-sample forecasts than those based on the symmetric distributed assumptions. Moreover, to test the explanatory power of the theoretical determinants we perform regression analysis on data from the pre-crisis (2004-2007) and crisis periods (2007-2010). We show that stock returns and equity volatility have statistically significant explanatory power on the iTraxx CDS index during each period, while the risk-free interest rate has no statistically significant effect on the iTraxx CDS index during the crisis period.

**JEL:** G10, G17, C22, C46

**Keywords:** iTraxx; CDS; Credit default swap; AR-GARCH; Skewed Student's t

## **1. Introduction**

The credit derivatives market has grown rapidly over the last five years. A credit default swap (CDS), one of the most important credit derivatives, is created to help banking and non-banking institutions efficiently diversify credit exposure. Principally, CDSs can be classified into two types: single-name and multi-name contracts. Single-name CDSs involve only one underlying entity whereas multi-name CDSs (e.g. CDS indices) comprise of a set of underlying entities in a portfolio pool. Our work focuses on CDS indices that are more efficient and liquid than holding a group of

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single-name CDS contracts as they allow investors to short or long the indices in order to adjust their credit risk exposure. The Dow Jones iTraxx family, one of the most liquid families of CDS indices, includes global companies with the exception of those based in North America. The most widely traded CDS index of the Dow Jones iTraxx family is the Dow Jones iTraxx Europe index, a portfolio made up of the 125 most liquid investment grade European companies in accordance with CDS volume traded. Each company has equally weighted credit exposure (notional principal).

Understanding the relationship between CDS spreads and its theoretical determinants has become an important issue. There are two pricing models for CDSs, structural form and reduced form models, of which our work focuses on the former to investigate the determinants of the most liquid iTraxx Europe 5-year CDS index spreads. Structural form models (or firm's value models) originated from the option pricing theory of Black and Scholes (1973), and were first formulated by Merton (1974), and subsequently extended by researchers such as Black and Cox (1976), Geske (1977), Leland and Toft (1996), Zhou (2001), Collin-Dufresne *et al.*(2001), and amongst others. The core concept of a structural form models is that an obligor defaults when the asset value of a company hits a specific threshold level.

Based on the structural form models, several earlier empirical studies have been applied to default swap markets. Collin-Dufresne *et al.*(2001), Longstaff *et al.* (2005), and Zhang *et al.* (2006) use linear regression to investigate the link between the key economic variables and bond yields. Recently, work has focused on extending earlier empirical studies on the iTraxx CDS market since iTraxx Europe CDS indices are more liquid than other relevant credit default instruments. Byström (2005), the first study extended to multi-name CDS spreads, tests the link between iTraxx spread changes and stock market using linear regression. This analysis shows that the iTraxx CDS market tends to be led by the stock market. Furthermore, he also demonstrates a significantly positive autocorrelation in the iTraxx market. Alexander and Kaeck (2008) and Ericsson *et al.* (2009) find that the changes in the CDS spreads are not only significantly affected by the firm's equity value, but also influenced by the equity volatility and risk-free interest rates. They observe that the higher the equity volatility, the higher the firm's value volatility, resulting in an upward trend in the CDS spread. Moreover, a low risk-free rate increases the default probability in the iTraxx CDS market. However, all these earlier empirical studies assume the error term of the linear regression model is normally distributed.

Financial time series are typified by volatility clustering behaviour; hence the autoregressive conditional heteroskedasticity (ARCH) model proposed by Engle (1982) and the generalized autoregressive conditional heteroskedasticity (GARCH) introduced by Bollerslev (1986) are very popular for modelling financial volatility.

These models are also being used to develop a more flexible and robust model to forecast financial time series in order to provide an explanation for volatility clustering and, implicitly, heavy-tailedness. However, numerous empirical studies show that financial time series often show skewed distribution conditionally as well as unconditionally, which implies that very large changes in returns occur with a higher frequency than under the symmetrically distributed assumption. A common assumption in applying standard GARCH models to financial time series is that the return series is conditionally symmetrically distributed (i.e. normally or student's t distributed), which are rarely able to accommodate the excess of skewness.

In this study the AR-GARCH model with the skewed student's t distribution proposed by Hansen (1994) is implemented to investigate the volatility clustering feature of the iTraxx Europe CDS index variation against its determinants. The skewed student's t distribution involves higher-order features of the conditional distribution, allowing the asymmetric behaviours of the error term to be captured. The symmetric student's t distribution can be seen as a special case in which the skewness parameter is set to zero. As more skewness is imposed, the skewed student's t distribution can be used to describe various combinations of asymmetric tail behaviours.

The rest of the paper is organized as follows. In Section 2, the theoretical determinants of CDS spreads are discussed. In Section 3, the AR-GARCH model with different distributed assumptions and economic methodology of volatility forecasting performance will be introduced. In Section 4, we summarize the empirical analyses and results using the AR-GARCH with different marginal density functions. Conclusions are presented in Section 5.

## **2. The Theoretical Determinants of CDS Spreads**

Theoretically, the main economic determinants of structural form pricing models are firm value, equity volatility, and the risk-free interest rate. Numerous structural form models have been extended from the basic structural form model suggested by Merton (1974), and all the extended studies are central to these three key determinants. Hence, in this study, we use linear regression analysis to investigate the relationship between the iTraxx spread changes with these three key variables, rather than including all systematic factors suggested by other extensive studies.

A decrease in the market value of underlying entity results in an increase in the default probability, since the underlying entity is approaching the default barrier. However, estimating the value of firm assets and liabilities is not easy because investors can not get all the information analogous to that of the company's managers. Since the changes in the firm's stock price can reflect on the condition of the firm's

operation, we use the Dow Jones Euro STOXX 600 index to represent the performance of the Euro zone equity market. The Dow Jones Euro STOXX 600 reflects the performance of the 600 largest market capitalized companies in the European economic region.

The variation of credit risk premia is an increasing function of equity volatility, since the likelihood of hitting the default barrier will increase while the fluctuation of firm value widens. The equity volatility can be estimated using either historical data or implied volatility based on stock options. Benkert (2004) demonstrates that explaining the variation in CDS spreads using implied volatility has a stronger impact than using historical volatility. However, considering the lack of traded options on some reference entities of the iTraxx Europe CDS index and illiquidity on some traded options, the implied volatility constructed by all available single-name underlying entities may not reflect all the current business risk in the European market. In this study, we choose the option-implied volatility index, the Dow Jones Euro VSTOXX 50 index, as a proxy of equity volatility instead. The Dow Jones Euro VSTOXX 50 index is based on Dow Jones Euro STOXX 50 options prices, which is designed to reflect near-term volatility market expectations by determining the implied volatility across all underlying equity options with a given time to maturity.

The theoretical arguments support that the credit default spread is inversely related to the risk-free interest rate (see Duffie, 1999). The risk-free rate determines the risk neutral drift in firm value, that is, an increase in the risk-free interest rate tends to lower the risk-neutral default probability. Houweling and Vorst (2005) use the swap rate as a proxy of the risk-free interest rate and find a stronger impact of the swap rate on the CDS market than that of the Treasury rate. Thus, in our approach we use the 1-year Euro Swap middle rates as a proxy for risk-free rates, which may provide near-term expectations on the European economic and credit environment.

### 3. Volatility Modelling and Evolution of Volatility Forecasts

#### 3.1 The AR-GARCH Model with Different Distributed Assumptions

An autoregressive (AR) model is chosen for the conditional mean to allow for possible autocorrelation in the lagged iTraxx index spreads. The conditional mean equation of a univariate time series  $y_t$  can be expressed as:

$$y_t = \sum_{i=1}^s A_i y_{t-i} + \varepsilon_t \quad (1)$$

$$\varepsilon_t = \sigma_t z_t, \quad z_t \sim N(0,1) \quad (2)$$

where  $z_t$  is a sequence of independently identically distributed random variables,

$\sigma_t$  is conditional variance, and  $\varepsilon_t$  denotes the error term of the time series. Therefore, the conditional variance equation of Bollerslev (1986) can be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2, \quad (3)$$

where  $\omega$ ,  $\alpha_i$  and  $\beta_j$  are non-negative integers with  $\sum_{i=1}^q \alpha_i + \sum_{j=1}^p \beta_j < 1$  to ensure the satisfaction with the sufficient stable condition of strictly stationary in the GARCH process. Moreover,  $p$  and  $q$  are also restricted to be non-negative integers.

The inability of the symmetric GARCH model to accommodate the excess of skewness in financial series is well-known. The skewed student's t density function proposed by Hansen (1994) has the additional benefit of including higher-order features of the conditional distribution. Therefore, the skewed student's t density function, present in Equation (4), will be used to account for the excess of skewness in this study:

$$g(t | \nu, \lambda) = \begin{cases} bc(1 + \frac{1}{\nu-2} (\frac{bt+a}{1-\lambda})^2)^{\frac{-(\nu+1)}{2}} & t < -\frac{a}{b} \\ bc(1 + \frac{1}{\nu-2} (\frac{bt+a}{1+\lambda})^2)^{\frac{-(\nu+1)}{2}} & t \geq -\frac{a}{b} \end{cases} \quad (4)$$

with  $a$ ,  $b$ , and  $c$  defined as:

$$a = 4\lambda c \left( \frac{\nu-2}{\nu-1} \right), \quad (5)$$

$$b^2 = 1 + 3\lambda^2 - a^2, \quad (6)$$

$$c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)\Gamma(\frac{\nu}{2})}}, \quad (7)$$

where  $\nu$  is the tail index and  $\lambda$  is the skewness parameter, which are used to control the different shape of the density function and  $\Gamma(\cdot)$  is the gamma function. The two shape parameters need to be restricted with  $\nu > 2$  and  $-1 < \lambda < 1$  respectively. If  $1 < \lambda < 0$  the density function skews toward to the right, conversely if  $0 > \lambda > -1$  the density function skews toward to the left. Furthermore, the skewed student's t distribution transforms to the normal distribution by setting  $\nu = \infty$  and  $\lambda = 0$  and to symmetric student's t distributions by setting  $\lambda = 0$ .

## 3.2 Evaluation of Volatility Forecasting Performance

### 3.2.1 Loss Functions

A common way to compare different forecast models is given by the minimization of a statistic loss function. AR-GARCH models describe the evolution of the conditional mean and conditional variance. Once the parameters of the AR-GARCH models have been estimated using in-sample period data, and conditional variance forecasts based on these estimates can be generated over the out-of-sample period. In order to evaluate the forecast accuracy of various competing models, statistical loss functions have to be used to measure forecast errors. Mean squared error (*MSE*) is the most commonly used loss function, however, the *MSE* criterion is very sensitive to outliers. Hence, a more robust loss function to possible presence of outliers, mean absolute error (*MAE*) is taken into account in this study. Moreover, the logarithmic loss (*LL*) function, proposed by Pagan and Schwert(1990), penalizes volatility forecasts asymmetrically in low volatility and high volatility periods, which can be used to deal with the second shortcoming of the *MSE* criterion. These three loss functions are defined as follows:

$$MSE = \frac{1}{n} \sum_{t=T+1}^{T+n} (h_t^2 - \sigma_t^2)^2, \quad (8)$$

$$MAE = \frac{1}{n} \sum_{t=T+1}^{T+n} |h_t - \sigma_t|, \quad (9)$$

$$LL = \frac{1}{n} \sum_{t=T+1}^{T+n} [\ln(h_t^2) - \ln(\sigma_t^2)]^2, \quad (10)$$

where  $n$  is the number of forecast data points,  $T$  is the entire in-sample period,  $h_t^2$  is forecast conditional variance at time  $t$ , and  $\sigma_t^2$  is realized conditional variance at time  $t$ .

### 3.2.2 Diebold-Mariano Test (*DM* Test)

Diebold and Mariano (1995) propose a test to evaluate the accuracy of  $L$ -steps ahead forecasts of one model in comparison to another. The null hypothesis of the test is that two competing models have equal forecast accuracy against the alternative hypothesis of significant difference in the accuracy of the two competing forecasts. The *DM* test is based on a  $t$ -test that  $E(\delta_t) = 0$  where  $E(\cdot)$  is the expectation operator,  $\delta_t = l_t^A - l_t^B$ ,  $l_t^A$ , and  $l_t^B$  are forecast errors of two competing models  $A$  and  $B$  respectively. The *DM* test  $t$  statistic can be expressed as:

$$DM = \frac{\bar{\delta}}{\sqrt{V(\bar{\delta})}} \quad (11)$$

where  $\bar{\delta}$  and  $V(\bar{\delta})$  is mean and variance of forecast errors over the forecasting period respectively. However,  $\delta_t$  can not be assumed to be uncorrelated. The *DM* test assumes that the autocorrelations of order  $L$  or higher are zero of  $\delta_t$ , so  $V(\bar{\delta})$  can be estimated asymptotically as:

$$V(\bar{\delta}) \approx \frac{1}{n} \left( \gamma_0 + 2 \sum_{j=1}^{L-1} \gamma_j \right) \quad (12)$$

where  $\delta_t$  is serially correlated for  $n > 1$  and  $\gamma_j$  denotes the  $j$ -th autocovariance of  $\delta_t$ .

## 4. Empirical Analysis

### 4.1 Data Description

Our data set on the most liquid iTraxx 5-year Europe CDS index, the STOXX 600 index, the VSTOXX 50 index, and the 1-year Euro swap rate consists of 1375 daily observations; it covers the period from August 16, 2004 to January 28, 2010. Although the iTraxx 5-year Europe CDS index was launched in June 2004, our data set lacks some trading day observations on the iTraxx CDS index before mid-August 2004. As a result, we use data from August 16, 2004 onward in this study. Since the new series of the iTraxx Europe CDS index is launched every six months in order to adjust the underlying companies, we use the most current series data of the iTraxx index at any point in time to ensure our analysis is always based on the most liquid underlying entities. The data set on 1-year Euro Swap rate is obtained from Datastream, the data for the STOXX 600 and the VSTOXX 50 indices are obtained from Stoxx Ltd<sup>1</sup>, and the iTraxx CDS index data is available from Markit Group<sup>2</sup>.

### 4.2 Descriptive Statistics

Before investigating the relationship between the iTraxx CDS index and the explanatory variables, we provide summary statistics for the iTraxx CDS index and the key variables. We consider the natural logarithmic returns of all the time series. In order to determine the presence of mean reversion in the logarithm of returns, the unit root tests (Augmented Dickey-Fuller test (ADF test)) can be used to verify the behaviour of the time series. The summary statistics and the unit test results are

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<sup>1</sup> [www.stoxx.com](http://www.stoxx.com)

<sup>2</sup> [www.markit.com](http://www.markit.com)

presented in Table 1.

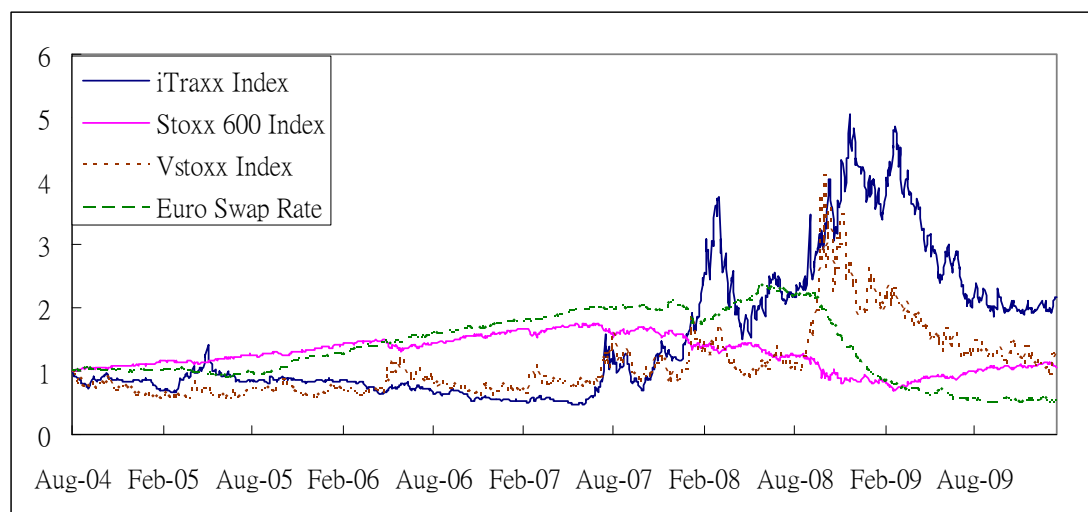
The results in Table 1 show that the distributions of log returns of the iTraxx index and its theoretical determinants are skewed. Moreover, the iTraxx CDS and the VSTOXX 50 indices are more volatile than the STOXX 600 index and the Euro swap rate. The ADF test is run on the level of the log returns of all indices. One can see from the results that all the null hypotheses under the ADF test can be rejected at the 5% level. The mean-reverting coefficients in the ADF test is statistically significantly negative, confirming the stationarity of periodically mean-reverting cycles.

**Table 1 Summary statistics of daily logarithmic returns**

	<b>iTraxx</b>	<b>STOXX600</b>	<b>VSTOXX 50</b>	<b>Euro Swap</b>
<b>Mean (%)</b>	0.0493	0.0025	0.0010	-0.0460
<b>Median (%)</b>	-0.1243	0.0732	-0.5146	0.0255
<b>Maximum (%)</b>	25.5365	9.4100	32.7675	5.5169
<b>Minimum (%)</b>	-26.9760	-7.9297	-19.8182	-5.0990
<b>Std. Dev. (%)</b>	3.76	1.35	5.60	1.05
<b>Skewness</b>	0.2985	-0.1358	0.9600	-0.1285
<b>Kurtosis</b>	10.5970	11.1317	6.5341	7.2760
<b>Jarque-Bera Test</b>	3326.9970*	3792.5717*	926.7308*	1051.3420*
<b>ADF Test</b>	-31.7771*	-38.6537*	-39.1200*	-32.7481*

\* denote significantly at 5% significance level.

**Figure 1. The iTraxx Europe 5-year index and its determinants, normalized to start at 1**



Byström (2005, 2006), and Alexander and Kaeck (2008) point out that the iTraxx Europe CDS index exhibits significant autocorrelation in its previous day's



changes, resulting in the inefficiency in the European 5-year CDS market. According to the results of the Ljung Box test,  $Q(20)$  and  $Q(30)$  are 61.8043 and 78.883 respectively, supporting earlier empirical studies. In order to deal with the autocorrelation problem, the effect of the lagged returns on previous day's iTraxx CDS spread will be included in the next section.

As seen in Figure 1, before 26 July 2007 the iTraxx CDS index demonstrates stable downward trend, and displays positive correlation with the VSTOXX 50 index and negative correlation with the Euro swap rate and the STOXX 600 index. After 26 July 2007, all key market variables and the iTraxx CDS index fluctuated dramatically. It's worth mentioning that the Euro swap rate seems not to be significantly negatively correlated to the iTraxx index after 26 July 2007.

### 4.3 Linear Regression Analysis

In this section, we use the following linear regression (Equation (13)) to analyze the dependence between the daily log returns of the iTraxx CDS spreads and daily log returns of the explanatory variables.

$$\Delta CDS_t = \theta_0 + \theta_1 \Delta S_t + \theta_2 \Delta V_t + \theta_3 \Delta CDS_{t-1} + \theta_4 \Delta r_t + \varepsilon_t, \quad (13)$$

The regression will be tested in three stages: the whole period, the pre-crisis period from August 16 2004 to July 25 2007, and the crisis period between July 26 2007 and January 28, 2010. Let  $\Delta CDS_t$  denote the logarithmic returns of the iTraxx CDS spreads,  $\Delta V_t$  denote logarithmic returns of the VSTOXX 50 index,  $\Delta r_t$  denote the logarithmic returns of the 1-year Euro swap rate, and  $\Delta S_t$  denote the logarithmic returns of the STOXX 600 index. The error term,  $\varepsilon_t$ , are i.i.d. random errors with zero mean and unit variance, and will be assumed to be normally, symmetric student's t and skewed student's t distributed respectively. The regression results are given in Table 2.

All the explanatory variables are statistically significant at the 5% level in the main and two sub-period regressions except for the Euro swap rate during the crisis period. Byström (2005, 2006) and Alexander and Kaeck (2008) discover significant predictive ability for lagged daily iTraxx spread changes by using previous day's iTraxx spreads, the results in Table 2 support this in that the European CDS market exhibits significant positive first-order autocorrelation. However, the impact of the positive first-order autocorrelation in the crisis period is not as sensitive as in the pre-crisis period.

The fluctuation of the iTraxx CDS index has the negative correlation with the lagged returns in the STOXX 600 index and the positive correlation with the lagged

returns in VSTOXX 50 index, which is consistent with previous investigations. The fluctuation of the iTraxx CDS index is more sensitive to lagged returns in the STOXX 600 and the VSTOXX 50 indices during the crisis period. The Euro swap rate has a significant effect on the iTraxx CDS spreads during the pre-crisis period only. Alexander and Kaeck (2008) indicate that the risk-free rate has no statistically significant impact on the iTraxx CDS spreads during the turbulent period, consistent with our finding in the crisis period. We also use the 1-year Euribor rate as the proxy of the risk-free interest rate and the results appear similar to those displayed in Table 2. However, the values of  $R^2$  for three testing periods are not as good as using the Euro swap rate. For this reason, we do not provide the results of using the 1-year Euribor rate in the study.

**Table 2 Regressions with all explanatory variables**

	Whole Period	Pre-crisis Period	Crisis Period
$\theta_0$	0.0003 (0.3834)	0.0008 (1.0945)	-0.0001 (-0.0904)
$\theta_1$	-1.2179 (-13.1930)	-0.6893 (-4.2615)	-1.1443 (-8.3488)
$\theta_2$	0.0891 (4.0382)	0.0739 (3.1437)	0.1615 (4.0849)
$\theta_3$	0.1579 (7.2260)	0.2972 (9.3642)	0.1239 (3.9387)
$\theta_4$	-0.2816 (-3.4609)	-0.4975 (-4.2336)	-0.1967 (-1.6513)
$R^2$	0.3470	0.2526	0.3886
Adjusted $R^2$	0.3451	0.2486	0.3846

Each column reports the estimates of the regression by Eq. (13), with t-statistics below in brackets.

Comparing the two sub-periods, the market factors have a stronger effect during the crisis period than in the pre-crisis period. The degree of explanation,  $R^2$ , can be used to measure how well the regression estimates the time series. In the crisis period, the iTraxx CDS spreads were more volatile and  $R^2$  is much higher than that in the pre-crisis period, which may indicate that these theoretical determinants have stronger explanatory power in the period of high volatility. Using the proxy of firm value suggested by Alexander and Kaeck (2008) in Equation (13), the values of  $R^2$  are around 31% for the whole period and 36% for the crisis period. The main difference is that Alexander and Kaeck (2008) generate an equally weighted stock portfolio comprised of most of the underlying entities from the iTraxx 5-year CDS Europe

index as the proxy firm value. However, the iTraxx CDS index only covers 125 investment grade European companies, which may not reflect in entirety the European economic situation. On the contrary, the STOXX 600 index can represent the performance of the Euro zone equity market, which may explain the daily fluctuation of the iTraxx CDS index better.

#### 4.4 Residual Analysis

The AR(1)-GARCH(1,1) model with the skewed student's  $t$  marginal distribution is used to estimate the behaviour of the regression residuals instead of the standard AR-GARCH models. In order to assess the practical relevance of this distribution, the comparison among the skewed student's  $t$  distribution, the normal and the symmetry student's  $t$  distributions are taken into account. In addition, the Ljung Box Q-Test, the ARCH-LM test, and the Pearson goodness-of-fit (GOF) test are used to address estimation risk. The Ljung Box Q-Test for squared standardized residuals detects any remaining serial correlation in the conditional variance equation. If the conditional variance equation is correctly specified, all Q-statistics of standardized residuals should be insignificant with no observable autocorrelation. The ARCH-LM tests the null hypothesis that there is no ARCH effect up to order  $q$  in the squared residuals. Moreover, we employ the Pearson goodness-of-fit test for standardized residuals as a statistical diagnostic in order to test the null hypothesis of the distribution of certain time series is consistent with a particular theoretical distribution.

Table 3 shows the AR(1)-GARCH(1,1) estimations and the tests results for the whole period. The AR(1)-GARCH(1,1) model can be adequate to capture the dynamics of the residual series of Equation (13). All the ARCH-LM tests are highly statistically significant at the 5% level, meaning the heteroskedasticity is removed. The results of the Ljung Box Q-statistic with 30 lags test accept the null hypothesis of no autocorrelation, meaning the AR(1)-GARCH (1,1) model is adequate to capture the conditional mean and the conditional variance. The estimated numbers of degrees of freedom  $\nu$  are statistically significant lying on 5.92 and 5.68, which indicate residual series of the whole period to be fat-tailed. The skewness parameter  $\lambda$  is positively significantly different from 0 at the 5% level, exhibiting the distributions of residual series seem to be skewed.

According to the Pearson goodness-of-fit statistics, the skewed student's  $t$  distribution is more suitable to capture the volatility clustering due to the insignificant Pearson goodness-of-fit statistics of the normally and the symmetric student's  $t$  distributed assumptions. Moreover, the Akaike information criteria (AIC) and the

log-likelihood values highlight the fact that AR(1)-GARCH(1,1) with skewed student's t marginal density function better estimate the residual time series than symmetric AR(1)-GARCH(1,1) model. Table 4 presents the Jarque-Bera and the Kolmogorov-Smirnov (KS) tests of the residual series for the whole period. The results show that the residual is neither normally distributed nor symmetric student's t distributed but skewed student's t distributed, supporting the theory that the AR(1)-GARCH(1,1) with skewed student's t marginal density function is more suitable to capture the volatility clustering feature of the regression residual.

**Table 3 Estimation results of AR(1)-GARCH(1,1) model for the whole period**

	Normal	Student's t	Skewed Student's t
<b>Const.</b>	0.0000 (164551.80)	0.0007 (5911.75)	0.0000 (646951.61)
<b>ARCH (1)</b>	0.2121 (74.6730)	0.1379 (224.8342)	0.1474 (147.3609)
<b>GARCH (1)</b>	0.7706 (412.9802)	0.8619 (802.2792)	0.8469 (998.6663)
$\nu$	$\infty$	5.9230 (6.9997)	5.6794 (9.6586)
$\lambda$	0	0	0.0732 (51.5312)
<b>AIC</b>	-6345.78	-6526.96	-6531.65
<b>Log-Lik</b>	3179.89	3271.48	3274.82
<b>GOF(10)</b>	26.0342 [0.0000]	47.8487 [0.0000]	4.2391 [0.1201]
<b>ARCH-LM Test</b>	0.0500 [0.8231]	0.3033 [0.5818]	0.0008 [0.9769]
<b>Ljung Box Q-Test</b>	17.0379 [0.9721]	14.8608 [0.9905]	15.0906 [0.9892]

Each column reports the estimates of the model defined in section 2.1, with t-statistics underneath in parentheses. The statistics of all tests are reported with p-value underneath in brackets. AIC and Log-Lik are the Akaike Information criterion and Log-Likelihood value

**Table 4 Distribution test of the residual series in the whole period**

	Jarque-Bera Test	KS (Symmetric t)	KS(Skewed t)
<b>Statistic</b>	8015.6756	0.0536	0.0193
<b>p-Value</b>	0.0010	0.0007	0.6799

As is typical of AR-GARCH model estimates, the sum of the coefficients on the lagged squared error and lagged conditional variance for the whole period with the skewed student's t marginal density function is very close to unity (approximate 0.9943). This implies that shocks to the conditional variance will be highly persistent. This can be demonstrated by considering the equations for forecasting the standardized regression residuals of the conditional variance using the AR(1)-GARCH(1,1) model based on skewed student's t distribution. A large sum of these coefficients implies that a large positive or a large negative change will lead to high future variance forecast for a protracted period.

#### 4.5 Out-of-Sample Forecast Evaluation

An out-of-sample forecast evaluation provides a powerful framework to evaluate the performances of competing models. Many empirical studies (e.g. Pagan and Schwert(1990), Davis and Kutan(2003), Alberg, Shalit and Rosef(2008), and so on) demonstrate that the asymmetric assumption of conditional variance provides a better in-sample fit and better out-of-sample predictive power. In this section, the whole data set is divided into an in-sample period from August 16, 2004 to September 12 2008 and an out-of-sample period from September 15 2008 to January 28 2010 for the forecasting evaluation.

Table 5 reports the out-of-sample forecast evaluation results using the statistical loss function from section 3.2.1 of this study together with various distributions. All of the loss functions clearly indicate that the AR(1)-GARCH(1,1) model with skewed student's t marginal density function performs best for all of the four forecast horizons, and the second best is based on the student's t distribution. In order to examine if the forecast accuracy among the three assumptions on distribution are significantly different, the *DM* test is adopted to check the statistical significance when all models are compared in pairwise.

Table 6 presents the *DM* test results using the skewed student's t distribution as the benchmark, and the comparison is carried out with regards to all the statistical loss functions discussed in section 3.2.1. As can be seen in Table 6, the skewed student's t distribution significantly outperforms the symmetric models at the 5% level. The signs of the *DM* statistics of the comparison between the skewed student's t distribution and the symmetric distributions are always significantly positive; indicating the loss of the skewed student's t is significantly lower than that of symmetric distributions. In other words, the AR(1)-GARCH(1,1) with skewed student's t marginal density function provides a more accurate volatility forecast, demonstrating the skewed student's t distributed assumption of regression residual

outperforms the symmetric distributed assumption for the forecast evaluation.

**Table 5 Out-of-sample forecast evaluation results for statistical loss functions: 1-,5-,10- and 20-day forecast horizons**

	Normal	Student's t	Skewed t
<i>1 day</i>			
MSE (%)	0.2912	0.2029	0.1901
MAE (%)	0.00001	0.00001	0.00000
LL (%)	6.1755	3.5690	3.2217
<i>5 days</i>			
MSE (%)	0.4894	0.3671	0.3462
MAE (%)	0.00003	0.00002	0.000018
LL (%)	18.0083	12.1626	11.2584
<i>10 days</i>			
MSE (%)	0.5956	0.4823	0.4583
MAE (%)	0.00004	0.00003	0.00003
LL (%)	25.5309	19.6858	18.5395
<i>20 days</i>			
MSE (%)	0.7769	0.6723	0.6466
MAE (%)	0.00007	0.00006	0.00005
LL (%)	41.6212	35.5922	34.0249

**Table 6 Diebold-Mariano Test**

	MSE		MAE		LL	
	Statistic	p-Value	Statistic	p-Value	Statistic	p-Value
<i>1 day</i>						
Normal v.s. Skewed t	3.6167	0.0006	14.4720	0.0000	9.7414	0.0000
Student's t v.s. Skewed t	3.0810	0.0036	4.9934	0.0000	6.6068	0.0000
<i>5 days</i>						
Normal v.s. Skewed t	2.6666	0.0117	10.9426	0.0000	7.0684	0.0000
Student's t v.s. Skewed t	2.5321	0.0165	6.2849	0.0000	7.6274	0.0000
<i>10 days</i>						
Normal v.s. Skewed t	2.7083	0.0105	7.4710	0.0000	4.5816	0.0000
Student's t v.s. Skewed t	2.6424	0.0125	6.4235	0.0000	6.4022	0.0000
<i>20 days</i>						
Normal v.s. Skewed t	2.0971	0.0446	4.2894	0.0000	2.4304	0.0211
Student's t v.s. Skewed t	2.7718	0.0088	5.7786	0.0000	3.7596	0.0004

## 5. Conclusion

This study investigates the linear relationship between theoretical determinants and the iTraxx Europe CDS index. The theoretical determinants of the iTraxx CDS index have a significant impact on the iTraxx CDS market, except for 1-year Euro Swap rate, which has no statistically significant effect on the iTraxx CDS market in the crisis period. This finding indicates that the risk-free rate yield curve has no statistically significant effect on the iTraxx CDS market in the period of high volatility. Moreover, all the explanatory variables in this study have higher explanatory power than previous empirical studies due to a better proxy of firm's value suggested in this study.

Furthermore, financial time series are known to be conditionally heteroskedastic, fat-tailed, and often skewed. To accommodate for this we further relaxed the assumption of time series of regression residuals from symmetric distributions to an asymmetric distribution. Our results show that volatility clustering features in the iTraxx CDS market can not be fully captured by using the standard AR(1)-GARCH(1,1) model. The AR(1)-GARCH(1,1) model with the skewed student's t marginal density function is much more suitable to capture mean-reverting conditionally heteroscedastic process for innovations. In addition, the sums of the coefficients on the lagged squared error and lagged conditional variance are very close to unity, indicating a large positive or a large negative change will lead to high future variance forecast for a protracted period.

Out-of-sample forecasting comparison is carried out by comprising 1-day, 5-day, 10-day and 20-day ahead volatility forecasts. The statistical loss functions demonstrate that AR(1)-GARCH(1,1) model with skewed student's t marginal density function outperforms the standard AR(1)-GARCH(1,1) models in forecasting volatility for all four forecast horizons. The Diebold-Mariano is applied to further examine whether the differences in the performances among three distributed assumptions are significant, and the results show that the AR(1)-GARCH (1,1) model based on the skewed student's t distributed assumption significantly outperforms other two competing models. Overall, the inclusion of AR(1)-GARCH(1,1) effects with fat-tailedness and skewness provides not only a better in-sample fit but also more accurate out-of-sample forecasts.

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